# Results

## Structural Equation Modeling (SEM)

### *Introduction*

A SEM model was conducted to determine whether the latent variables (Democracy1960, INdustry1960, and Democracy1965) adequately describe the data. Bootstrapping was performed using a maximum of 100 iterations to determine the standard errors for the parameter estimates.

### *Assumptions*

**Multivariate Outliers.** Influential points were identified in the data by calculating Mahalanobis distances and comparing them with the quantiles of a $\chi^2$ distribution (Newton & Rudestam, 2012). An outlier was defined as any Mahalanobis distance that exceeds 27.88, the .999 quantile of a $\chi^2$ distribution with 9 degrees of freedom (Kline, 2015). There were no outliers detected in the model.

**Multicollinearity.** Although variables should be correlated with one another to be considered suitable for factorization, variables that are too highly correlated can cause problems in SEM. To assess multicollinearity, the squared multiple correlations were inspected and the determinant of the correlation matrix was calculated. Any variable with an $R^2 > .90$ can contribute to multicollinearity in the SEM model (Kline, 2015). Variables that exhibit high multicollinearity should either be removed from the analysis or combined as a composite variable. There were no variables that had an $R^2 > .90$. Another assessment for multicollinearity is to assess the determinant of the data's correlation matrix. A determinant that is $\leq 0.00001$ indicates that multicollinearity exists in the data (Field, 2017). The value of the determinant for the correlation matrix was 0.0007, indicating that there was no multicollinearity in the data.

### *Results*

First, the reliability of the analysis was tested based on the sample size used to construct the model. Next, the results were evaluated using the Chi-square goodness of fit test and fit indices. Lastly, the squared multiple correlations ($R^2$) for each endogenous variable were examined. The results of the SEM model are presented in Table 1. The correlations between the latent variables are presented in Table 2. The node diagram is shown in Figure 1.

**Table 1**

*Unstandardized Loadings (Standard Errors), Standardized Loadings, and Significance Levels for Each Parameter in the SEM Model (N = 75)*

| Parameter Estimate | Unstandardized | Standardized | *p* |
|---|---|---|---|
| Loadings | | | |
| Democracy1960 → press1960 | 1.00(0.00) | 0.88 | -- |
| Democracy1960 → freedom1960 | 1.24(0.16) | 0.72 | < .001 |
| Democracy1960 → fairness1960 | 1.02(0.15) | 0.71 | < .001 |
| INdustry1960 → gdp1960 | 1.00(0.00) | 0.92 | -- |
| INdustry1960 → consumption1960 | 2.19(0.16) | 0.97 | < .001 |
| INdustry1960 → employment1960 | 1.82(0.15) | 0.87 | < .001 |
| Democracy1965 → press1965 | 1.00(0.00) | 0.81 | -- |
| Democracy1965 → fairness1965 | 1.20(0.15) | 0.77 | < .001 |
| Democracy1965 → freedom1965 | 1.13(0.18) | 0.70 | < .001 |
| Regressions | | | |
| Democracy1960 → Democracy1965 | 0.85(0.09) | 0.98 | < .001 |
| INdustry1960 → Democracy1965 | 0.90(0.23) | 0.30 | < .001 |
| Errors | | | |
| Error in INdustry1960 | 0.45(0.08) | 1.00 | < .001 |
| Error in Democracy1965 | -0.22(0.33) | -0.06 | .494 |
| Error in press1960 | 1.57(0.37) | 0.23 | < .001 |
| Error in freedom1960 | 7.39(1.59) | 0.48 | < .001 |
| Error in fairness1960 | 5.23(1.27) | 0.49 | < .001 |
| Error in gdp1960 | 0.08(0.02) | 0.16 | < .001 |
| Error in consumption1960 | 0.12(0.07) | 0.05 | .096 |
| Error in employment1960 | 0.46(0.09) | 0.24 | < .001 |
| Error in press1965 | 2.05(0.50) | 0.34 | < .001 |
| Error in freedom1965 | 5.22(0.98) | 0.51 | < .001 |
| Error in fairness1965 | 3.95(0.83) | 0.41 | < .001 |

| | | | |
|---|---|---|---|
| Error in Democracy1960 | 5.22(0.78) | 1.00 | < .001 |

*Note.* $\chi^2(25) = 49.22$, $p = .003$; -- indicates the statistic was not calculated due to parameter constraint.

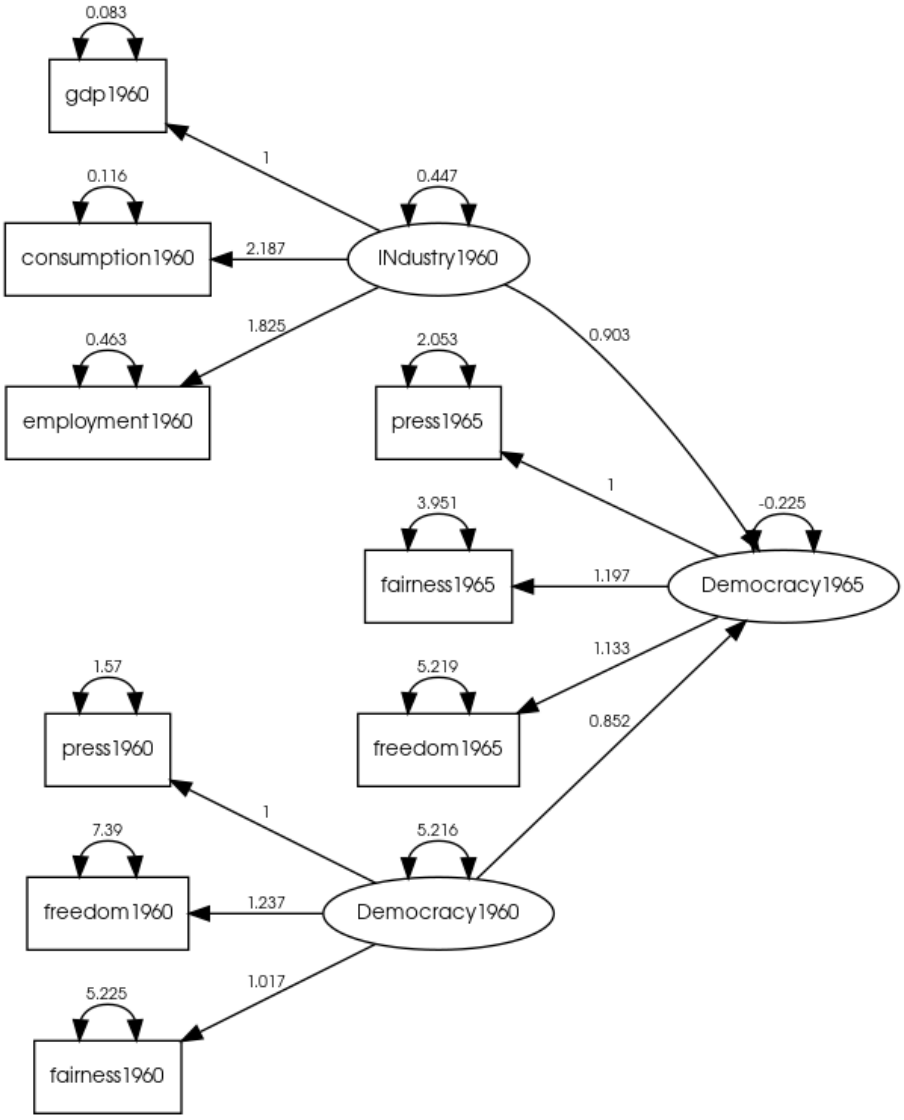
**Table 2**

*Correlation Table for the Latent Variables*

| Variable | Democracy1960 | INdustry1960 | Democracy1965 |
|---|---|---|---|
| Democracy1960 | 1.00 | -- | -- |
| INdustry1960 | 0.00 | 1.00 | -- |
| Democracy1965 | .98 | .30 | 1.00 |

**Figure 1**

*Node diagram for the SEM model*




**Evaluating sample size.** Factor analysis requires a large sample size to construct repeatable and reliable factors. A variety of authors suggest different benchmarks to determine sufficient sample size for SEM. Some authors use benchmarks based on overall sample size. A common rule of thumb for determining sufficient sample size is 300 observations (Tabachnick & Fidell, 2013; Comrey & Lee, 2013). Other authors use the ratio (*N:q*) of overall sample size to the number of free parameter estimates (latent variable, indicator, variance, covariance or any regression estimates) included in the model. Kline (2015) recommends that the *N:q* ratio should be about 20 to 1. Schreiber et al. (2006) suggest that the consensus for a sufficient *N:q* ratio is 10:1. On the lower end of the ratio, Bentler and Chou (1987) suggest that an acceptable *N:q* ratio

is 5:1. The participant to item ratio for this analysis was approximately 3 to 1, where sample size was 75 and the number of variables included was 20. According to the *N:q* ratio rule-of-thumb, the given sample size is insufficient for SEM. Consider collecting more data, as the results may be unreliable.

**Model fit.** There are a variety of ways to measure if the SEM model adequately describes the data. The Chi-square statistic is the most popular statistic used to measure model fit. Besides the Chi-square statistic, fit indices are also used to help researchers determine if the factor analysis model fits the data properly. Along with the Chi-square goodness of fit test, the following fit indices were used to assess the model fit: root mean square error of aproximation (RMSEA), comparative fit index (CFI), Tucker-Lewis index (TLI) and standardized root mean square residual (SRMR).

***Fit indices.*** The TLI was less than .95, TLI = 0.93, which is indicative of a poor model fit (Hooper et al., 2008). The CFI was greater than .95, CFI = 0.95, suggesting that the model fit the data well (Hooper et al., 2008). The RMSEA index was greater than .10, RMSEA = 0.11, 90% CI = [0.07, 0.16], which is indicative of a poor model fit (Hooper et al., 2008). The SRMR was greater than .08, SRMR = 0.17, which implies that the model fits the data poorly (Hooper et al., 2008). The fit indices are presented in Table 3.

***Goodness of fit test.*** A Chi-square goodness of fit test was conducted to determine if the SEM model fits the data adequately. It is standard practice for SEM to include the Chi-square test. However, this test is sensitive to sample size, which causes the test to almost always reject the null hypothesis and indicate a poor model fit when the sample size is large (Hooper et al., 2008). The results of the Chi-square goodness of fit test were not significant, $\chi^2(25) = 49.22$, $p = .003$, suggesting that the model fit the data well.

**Table 3**

*Fit Indices for the SEM model*

| NFI | TLI | CFI | RMSEA | SRMR |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| 0.91 | 0.93 | 0.95 | 0.11 | 0.17 |

*Note.* RMSEA 90% CI = [0.07, 0.16]; -- indicates that the statistic could not be calculated.

**Squared multiple correlations.** The regressions in the model can be assessed by examining the $R^2$ value of each endogenous variable. The $R^2$ value identifies how much the endogenous variable is explained by the regressions in the model. An $R^2$ value $\leq .20$ suggests the endogenous variable is not adequately explained by the regression(s) in the model and all regressions for that endogenous variable should be considered for removal from the model (Hooper et al., 2008). There were no endogenous variables with $R^2$ values $\leq .20$. The $R^2$ values, along with the error variances for each endogenous variable are presented in Table 4.

**Table 4**

*Estimated Error Variances and $R^2$ Values for Each Endogenous Variable in the SEM model.*

| Endogenous Variable | Standard Error | $R^2$ |
|---|---|---|
| Democracy1965 | -0.22 | -- |
| press1960 | 1.57 | .77 |
| freedom1960 | 7.39 | .52 |
| fairness1960 | 5.23 | .51 |
| gdp1960 | 0.08 | .84 |
| consumption1960 | 0.12 | .95 |
| employment1960 | 0.46 | .76 |
| press1965 | 2.05 | .66 |
| freedom1965 | 5.22 | .49 |
| fairness1965 | 3.95 | .59 |

*Note.* -- indicates the statistic could not be calculated.

**Interpretations for regressions.** The regressions were examined based on an alpha value of .05. Democracy1960 significantly predicted Democracy1965, $B = 0.85$, $z = 9.70$, $p < .001$, indicating a one-unit increase in Democracy1960 will increase the expected value of Democracy1965 by 0.85 units. INdustry1960 significantly predicted Democracy1965, $B = 0.90$, $z = 3.87$, $p < .001$, indicating a one-unit increase in INdustry1960 will increase the expected value of Democracy1965 by 0.90 units.

## References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# Glossaries

## Structural Equation Modeling

Structural Equation Modeling is a multivariate statistical technique to assess how well a specified number of scale variables represent the given constructs with regressions included in the model. SEM has similarities to exploratory factor analysis, but in SEM one specifies which scale variables load onto a certain construct. The SEM model is commonly used to determine mediating or indirect effects for relationships between observed or latent variables. SEM assesses whether the hypothesized underlying constructs fit the data well.

**Adjusted Goodness-of-Fit Index (AGFI):** An absolute fit index used for SEM to assess model fit. Values greater than .90 indicate a good fit and values less than .90 indicate a poor fit. This fit index is not recommended to be used on its own.

**Chi-Squared Statistic ($\chi^2$):** A test statistic based on the $\chi^2$ distribution. Used with the *df* to calculate a *p*-value.

**Comparative Fit Index (CFI):** A relative fit index used for SEM to assess model fit. Values greater than .95 indicate a good fit, values between .90 and .95 indicate an acceptable fit, and values less than .90 indicate a poor fit.

**Covariance:** A measurement that indicates the degree that two variables are linearly related to one another. Used in SEM and SEM model specifications to imply if two latent variables or indicator variables are correlated with each other.

**Degrees of Freedom (*df*):** Refers to the number of values used to compute a statistic; used in conjunction with a test-statistic to calculate the *p*-value.

**Determinant:** A value calculated from a square (n × n) matrix with useful mathematical properties.

**Direct Effect:** The effect that an independent variable has on the dependent variable without including any indirect effects.

**Endogenous Variable:** A variable in a structural equation modeling system that is affected by other variables in the model.

**Error Variance:** Indicates how much variance of a given parameter that does not contribute to the latent variable. Also referred to as the residual variance.

**Fit Index:** A value that is a measure of how well factor analysis model fits the data. The most common are: root mean square error of approximation, standardized root mean square residual (RMSEA), comparative fit index (CFI), and the Tucker-Lewis index (TLI).

**Goodness-of-Fit Index (GFI):** An absolute fit index used for SEM to assess model fit. Values greater than .95 indicate a good fit, values between .90 and .95 indicate an acceptable fit, and values less than .90 indicate a poor fit. This fit is not recommended to assess model fit.

**Indicator Variable:** A variable used to create a latent variable. Also referred to as the observed variable.

**Indirect Effect:** The effect that an independent variable has on the dependent variable through a mediating variable. The indirect shows the degree to which mediation occurs.

**Latent Variable:** A set of observed scale variables that have strong relationships with one another or have a similar pattern. Also referred to as a factor.

**Modification Indices:** A value that estimates the decrease in the Chi-square statistic for an improved model fit. Commonly used with CFA and SEM to respecify the model.

**Multicollinearity:** A state of very high intercorrelations or inter-associations among a set of variables.

**Tucker-Lewis Index (TLI):** A relative fit index used for SEM to assess model fit. Values greater than .95 indicate a good fit and values less than .95 indicate a poor fit, although .80 has been used as a cut off to indicate adequate fit.

**Normed-Fit Index (Bentler and Bonnet Index or NFI):** A relative fit index used for SEM to assess model fit. Values greater than .95 indicate a good fit, values between .90 and .95 indicate an adequate fit, and values less than .90 indicate a poor fit.

**Probability Value (*p*):** The probability of observing the test statistic under the null hypothesis.

**Root Mean Square Error of Approximation (RMSEA):** An absolute fit index for SEM to assess model fit. Values less than .08 indicate a good fit, values between .08 and .10 indicate an acceptable fit, and values greater than .10 indicate a poor fit.

**Squared Multiple Correlations ($R^2$):** Used in SEM to estimate each variable's communality. Also, referred to as $R^2$ in multiple linear regression. A value from 0 to 1 that shows the fraction of variance explained.

**Standard Error of Loading:** How much the unstandardized loading is expected to vary.

**Standardized Loading:** Ranges from -1 to 1, gives the strength of the relationship between the indicator variable and latent variable. Interpreted like a correlation.

**Standardized Root Mean Square Residual (SRMR):** An absolute fit index for SEM to assess model fit. Values less than .05 indicate a good fit, values between .05 and .08 indicate an acceptable fit, and values greater than .08 indicate a poor fit.

**Total Effect:** The effect that an independent variable has on the dependent variable including the direct effect and all indirect effects. The full effect that an independent variable has on the dependent variable including any mediation terms.

**Unstandardized Loading:** The slope of the indicator variable with the latent variable. Treated like a Beta coefficient in regression.

## Raw Output

### Structural Equation Modeling

Included Variables:
press1960, freedom1960, fairness1960, press1965, freedom1965, fairness1965, gdp1960, consumption1960, and employment1960

Sample Size (Complete Cases):
N = 75

Multivariate Outliers:
Mahalanobis distances vs. .999 quantile of $\chi^2$ distribution ($\chi^2(9) = 27.877$)

Outliers (Row Number in Data Set):
No Outliers

Multicollinearity:
Determinant of the Correlation Matrix $7.302 \times 10^{-04}$

Squared Multiple Correlations ($R^2$):

| Variable | $R^2$ |
|---|---|
| press1960 | 0.720 |
| freedom1960 | 0.569 |
| fairness1960 | 0.544 |
| press1965 | 0.682 |
| freedom1965 | 0.614 |
| fairness1965 | 0.627 |
| gdp1960 | 0.831 |
| consumption1960 | 0.857 |
| employment1960 | 0.741 |

Modification Indices:

| Parameter | Modification Indices | Parameter Change |
|---|---|---|
| freedom1960 ↔ freedom1965 | 14.832 | 3.238 |
| Democracy1960 ↔ INdustry1960 | 9.419 | 0.599 |
| fairness1960 ↔ freedom1965 | 6.249 | -1.757 |
| press1960 ↔ fairness1960 | 5.026 | 1.207 |
| press1960 ↔ fairness1965 | 4.152 | -0.977 |
| press1960 ↔ gdp1960 | 3.723 | 0.107 |
| freedom1960 ↔ gdp1960 | 3.238 | -0.191 |
| gdp1960 ↔ press1965 | 2.324 | 0.0879 |

| | | |
|---|---|---|
| fairness1960 ↔ fairness1965 | 1.905 | 0.887 |
| press1965 ↔ freedom1965 | 1.811 | -0.660 |

Estimated Error Variances and $R^2$ Values for Each Endogenous Variable:

| Endogenous Variable | Standard Error | $R^2$ |
|---|---|---|
| Democracy1965 | -0.225 | -- |
| press1960 | 1.570 | 0.769 |
| freedom1960 | 7.390 | 0.519 |
| fairness1960 | 5.225 | 0.508 |
| gdp1960 | 0.0834 | 0.843 |
| consumption1960 | 0.116 | 0.949 |
| employment1960 | 0.463 | 0.763 |
| press1965 | 2.053 | 0.657 |
| freedom1965 | 5.219 | 0.491 |
| fairness1965 | 3.951 | 0.588 |

Note: -- indicates the statistic could not be calculated.

Model Fit Indices:

| GFI | AGFI | NFI | TLI | CFI | RMSEA | SRMR |
|---|---|---|---|---|---|---|
| 0.877 | 0.778 | 0.909 | 0.931 | 0.952 | 0.114 | 0.171 |

Chi-square Goodness of Fit Test: $\chi^2(25) = 49.221$, $p = .003$; RMSEA 90% CI = [0.0656, 0.160]

Unstandardized Loadings (Standard Errors), Standardized Loadings, and Significance Levels for Each Parameter in the SEM Model (N = 75):

| Parameter Estimate | Unstandardized | Standardized | p |
|---|---|---|---|
| Loadings | | | |
| Democracy1960 → press1960 | 1.000(0.00000) | 0.877 | -- |
| Democracy1960 → freedom1960 | 1.237(0.164) | 0.721 | $5.307 \times 10^{-14}$ |
| Democracy1960 → fairness1960 | 1.017(0.145) | 0.713 | $2.550 \times 10^{-12}$ |
| INdustry1960 → gdp1960 | 1.000(0.00000) | 0.918 | -- |
| INdustry1960 → consumption1960 | 2.187(0.158) | 0.974 | 0.00000 |
| INdustry1960 → employment1960 | 1.825(0.154) | 0.873 | 0.00000 |
| Democracy1965 → press1965 | 1.000(0.00000) | 0.810 | -- |
| Democracy1965 → fairness1965 | 1.197(0.148) | 0.767 | $6.661 \times 10^{-16}$ |
| Democracy1965 → freedom1965 | 1.133(0.183) | 0.701 | $5.757 \times 10^{-10}$ |
| | | | |
| Regressions | | | |

| | | | |
|---|---|---|---|
| Democracy1960 → Democracy1965 | 0.852(0.0879) | 0.982 | 0.00000 |
| INdustry1960 → Democracy1965 | 0.903(0.234) | 0.305 | $1.098 \times 10^{-04}$ |
| | | | |
| Errors | | | |
| Error in INdustry1960 | 0.447(0.0817) | 1.000 | $4.631 \times 10^{-08}$ |
| Error in Democracy1965 | -0.225(0.329) | -0.0572 | 0.494 |
| Error in press1960 | 1.570(0.372) | 0.231 | $2.473 \times 10^{-05}$ |
| Error in freedom1960 | 7.390(1.588) | 0.481 | $3.243 \times 10^{-06}$ |
| Error in fairness1960 | 5.225(1.270) | 0.492 | $3.903 \times 10^{-05}$ |
| Error in gdp1960 | 0.0834(0.0178) | 0.157 | $2.652 \times 10^{-06}$ |
| Error in consumption1960 | 0.116(0.0695) | 0.0514 | 0.0956 |
| Error in employment1960 | 0.463(0.0869) | 0.237 | $1.010 \times 10^{-07}$ |
| Error in press1965 | 2.053(0.504) | 0.343 | $4.622 \times 10^{-05}$ |
| Error in freedom1965 | 5.219(0.980) | 0.509 | $9.980 \times 10^{-08}$ |
| Error in fairness1965 | 3.951(0.826) | 0.412 | $1.705 \times 10^{-06}$ |
| Error in Democracy1960 | 5.216(0.780) | 1.000 | $2.222 \times 10^{-11}$ |

Note: -- indicates the statistic was not calculated due to parameter constraint.